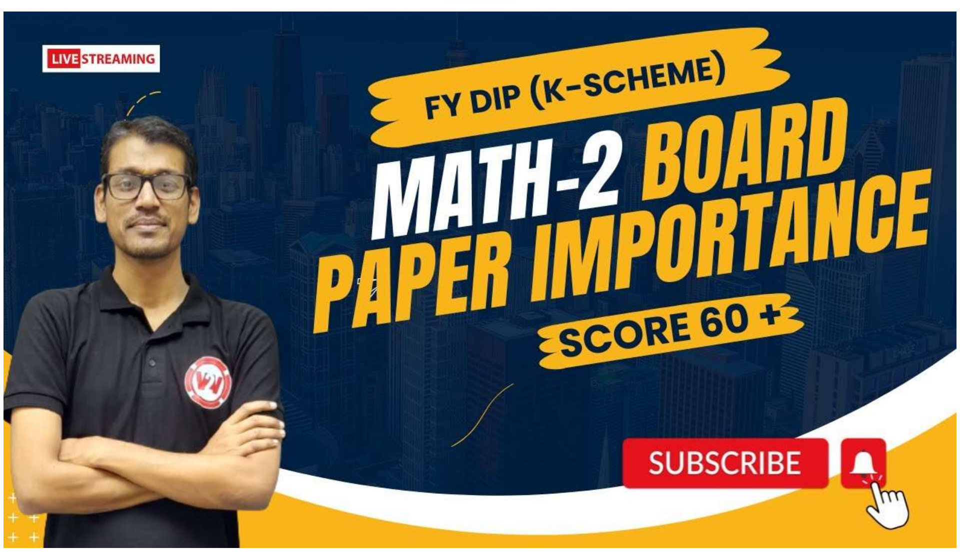
YouTube Lecture Links & Notes : K Scheme (Last Hour Revision Notes)

Topic : Integration : Lecture 1: YT Link: https://www.youtube.com_Maths2 : Lect 1
Topic : Integration : Lecture 2: YT Link: https://www.youtube.com_Maths2 : Lect 2
Topic : Differential Equation : Lecture 3: YT Link: https://www.youtube.com_Maths2:Lect 3
Topic : Numerical Method : Lecture 4: YT Link: https://www.youtube.com_Maths2 : Lect 4
Topic : Probability : Lecture 5: YT Link: https://www.youtube.com_Maths2 : Lect 5



Integration

1) $\int 0 \cdot dx = \text{constant}$
2) $\int 1\, dx = x + c$
3) $\int x^n dx = \frac{x^{n+1}}{n+1} + c$
4) $\int a^x dx = \frac{a^x}{\log_e a} + c$
5) $\int e^x dx = e^x + c$
6) $\int \frac{1}{x} dx = \log x + c$
7) $\int \frac{1}{ax+b} dx = \log|ax+b| \cdot \frac{1}{a} + c$
8) $\int \log x \, dx = x \log x - x + c$
9) $\int x e^x dx = e^x(x-1) + c$
10) $\int k x^n dx = k \int x^n dx$
11) $\int \sin x \, dx = -\cos x + c$
12) $\int \cos x \, dx = \sin x + c$
13) $\int \tan x \, dx = \log|\sec x| + c$
14) $\int \cot x \, dx = \log|\sin x| + c$
15) $\int \sec x \, dx = \log|\sec x + \tan x| + c$
16) $\int \operatorname{cosec} x \, dx = \log|\operatorname{cosec} x - \cot x| + c$
17) $\int \sec^2 x \, dx = \tan x + c$
18) $\int \operatorname{cosec}^2 x \, dx = -\cot x + c$
19) $\int \sec x \cdot \tan x \, dx = \sec x + c$
20) $\int \operatorname{cosec} x \cdot \cot x \, dx = -\operatorname{cosec} x + c$ //

* $\int \frac{f'(x)}{f(x)} dx = \log|f(x)| + c$,

$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + c$

$\int \frac{-1}{\sqrt{1-x^2}} dx = \cos^{-1} x + c$

$\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

$\int \frac{-1}{1+x^2} dx = \cot^{-1} x + c$

$\int \frac{x}{x\sqrt{x^2-1}} dx = \sec^{-1} x + c$

$\int \frac{-x}{x\sqrt{x^2-1}} dx = \operatorname{cosec}^{-1} x + c$

1 sum
4 Marks

* $\int \frac{1}{x^2+a^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c$
* $\int \frac{1}{x^2-a^2} dx = \frac{1}{2a} \cdot \log\left|\frac{x-a}{x+a}\right| + c$
* $\int \frac{1}{a^2-x^2} dx = \frac{1}{2a} \cdot \log\left|\frac{a+x}{a-x}\right| + c$

$$\int u * v \, dx = u \int v \, dx - \int \left[\frac{d}{dx} u \cdot \int v \, dx\right] dx$$

L = Logarithmic = $\log x$
I = inverse = $\sin^{-1} x \,/\, \tan^{-1} x \,/\, \cos^{-1} x$ - - -
A = algebric = $1, x, x^2$ - -
T = trignometric = $\sin x \,/\, \cos x \,/\, \tan x$ - - - -
E = exponential = $e^x \,/\, e^{2x} \,/\, e^{3x}$ - - -

→ $\text{Area} = \int_a^b y \cdot dx$

→ $\text{Area} = \int_a^b (y_1 - y_2) dx$

* $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$

Derivative

$\frac{d}{dx}\text{constant} = 0$

$\frac{d}{dx} x^n = n \cdot x^{n-1}$

$\frac{d}{dx} k x^n = k \cdot \frac{d}{dx} x^n$

$\frac{d}{dx} a^x = a^x \cdot \log_e a$

$\frac{d}{dx} e^x = e^x$

$\frac{d}{dx} \log x = \frac{1}{x}$

$\frac{d}{dx} \sin x = \cos x$

$\frac{d}{dx} \cos x = -\sin x$

$\frac{d}{dx} \tan x = \sec^2 x$

$\frac{d}{dx} \cot x = -\operatorname{cosec}^2 x$

$\frac{d}{dx} \sec x = \sec x \cdot \tan x$

$\frac{d}{dx} \operatorname{cosec} x = -\operatorname{cosec} x \cdot \cot x$

* $\frac{d}{dx}(u \pm v \mp w) = \frac{d}{dx} u \pm \frac{d}{dx} v \mp \frac{d}{dx} w$

* Product rule

$\frac{d}{dx}(u \cdot v) = u \cdot \frac{d}{dx} v + v \cdot \frac{d}{dx} u$

* division Rule

$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \cdot \frac{d}{dx} u - u \cdot \frac{d}{dx} v}{v^2}$

* Chain Rule

$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$

$\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx} \cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$

$\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$

$\frac{d}{dx} \cot^{-1} x = \frac{-1}{1+x^2}$

$\frac{d}{dx} \sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$

$\frac{d}{dx} \operatorname{cosec}^{-1} x = \frac{-1}{|x|\sqrt{x^2-1}}$

$$\frac{a}{0} = \infty$$

$$\frac{0}{a} = 0$$

$$a^0 = 1$$

$$a^m \times a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{m \times n}$$

$$a^x \times b^x = (ab)^x$$

$$\frac{1}{x^1} = x^{-1}$$

$$\frac{1}{x^3} = x^{-3}$$

$$\frac{1}{x^{7/2}} = x^{-7/2}$$

$$\frac{1}{x^{-9/4}} = x^{9/4}$$

$$\sqrt{x} = x^{1/2}$$

$$\sqrt[3]{x} = x^{1/3}$$

$$4\sqrt{x} = 4 \cdot x^{1/2}$$

Q. $I = \int \frac{1}{3x+4}\,dx$

$\Rightarrow \int \frac{1}{ax+b}\,dx = \log|ax+b| \cdot \frac{1}{a} + C$

$\therefore I = \log|3x+4| \cdot \frac{1}{3} + C$ //

Q. $I = \int \frac{1}{4-5x}\,dx$

$\int \frac{1}{ax+b}\,dx = \log|ax+b| \cdot \frac{1}{a} + C$

$\therefore I = \log|4-5x| \cdot \frac{1}{(-5)} + C$ //

$\sin 2A = 2\sin A \cdot \cos A$

$= \dfrac{2\tan A}{1+\tan^2 A}$

$\cos 2A = \cos^2 A - \sin^2 A$

$= 2\cos^2 A - 1$

$= 1 - 2\sin^2 A$

$= \dfrac{1-\tan^2 A}{1+\tan^2 A}$

$\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

$\sin 3A = 3\sin A - 4\sin^3 A$

$\cos 3A = 4\cos^3 A - 3\cos A$

$\sin^2\theta + \cos^2\theta = 1$

$1+\tan^2\theta = \sec^2\theta$

$1+\cot^2\theta = \operatorname{cosec}^2\theta$

Q. $I = \int (a^n + a^x + x^n + e^x)\,dx$

$\Rightarrow \quad = \int a^n dx + \int a^x dx + \int x^n dx + \int e^x dx$

$= a^n \int 1 \cdot dx + \dfrac{a^x}{\log_e a} + \dfrac{x^{n+1}}{n+1} + e^x + C$

$I = a^n (x) + \dfrac{a^x}{\log_e a} + \dfrac{x^{n+1}}{n+1} + e^x + C$ //

Q. $I = \int (\underline{\underline{a}}^{4x} + e^{5x} + x^6 + 7)\,dx$

$= \int a^{4x} dx + \int e^{5x} \cdot dx + \int x^6 dx + \int 7 \cdot dx$

$\int a^x dx = \dfrac{a^x}{\log_e a} + c$

$\int a^{4x} dx = \dfrac{a^{4x}}{\log_e a} \cdot \dfrac{1}{4} + c$

$\int e^x dx = e^x + c$

$\int e^{5x} dx = e^{5x} \cdot \dfrac{1}{5} + c$

$\int x^n dx = \dfrac{x^{n+1}}{n+1} + c$

$\int x^6 dx = \dfrac{x^{6+1}}{6+1} + c$

$= \dfrac{x^7}{7} + c$ //

$\int 7 \cdot dx = 7 \int 1 \cdot dx$

$= 7x + c$ //

$\therefore I = \dfrac{a^{4x}}{\log_e a} \cdot \dfrac{1}{4} + e^{5x} \cdot \dfrac{1}{5} + \dfrac{x^7}{7} + 7x + C$ //

Q. $I = \int x(x-1)^2 dx$

$\Rightarrow$ $(a-b)^2 = a^2 - 2ab + b^2$

$(x-1)^2 = x^2 - 2x \times 1 + 1^2$

$(x-1)^2 = x^2 - 2x + 1$

$\therefore I = \int x(x^2 - 2x + 1) \cdot dx$

$= \int (x^3 - 2x^2 + x) dx$

$= \int x^3 dx - \int 2x^2 dx + \int x^1 dx$

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$

$\therefore I = \frac{x^{3+1}}{3+1} - 2\int x^2 dx + \frac{x^{1+1}}{1+1} + C$

$I = \frac{x^4}{4} - 2\frac{x^3}{3} + \frac{x^2}{2} + c$ //

Q. $I = \int \sin^2 x \, dx$

$= \int \frac{1}{2}(1 - \cos 2x) dx$

$= \frac{1}{2}\int (1 - \cos 2x) dx$

$= \frac{1}{2}\left[\int 1 \cdot dx - \int \cos 2x \, dx\right]$

$= \frac{1}{2}\left[x - \sin 2x \cdot \frac{1}{2}\right] + C$ //

$\cos 2A = \cos^2 A - \sin^2 A$

$\cos 2A = 1 - 2\sin^2 A$

$2\sin^2 A = 1 - \cos 2A$

$\sin^2 A = \frac{1}{2}(1 - \cos 2A)$

$\int \cos x \, dx = \sin x + C$

$\int \cos 2x \, dx = (\sin 2x) \cdot \frac{1}{2} + C$

Q. $I = \int \frac{x}{(x+2)(x+3)} dx$

$$\Rightarrow \frac{x}{(x+2)(x+3)} = \frac{A}{(x+2)} + \frac{B}{(x+3)}$$

$$x = A(x+3) + B(x+2) \quad \text{——} (1)$$

$x+3=0$, $x=-3$ } eqn (1) $\Rightarrow$ $(-3) = A[(-3)+3] + B[(-3)+2]$

$$(-3) = A(0) + B(-1)$$

$$(-3) = B(-1)$$

$$\therefore \frac{(-3)}{(-1)} = \boxed{B=3}$$

$x+2=0$, $x=-2$ } eqn (1) $\Rightarrow$ $(-2) = A[(-2)+3] + B[(-2)+2]$

$$(-2) = A[1] + B(0)$$

$$\boxed{(-2) = A}$$

$$\frac{x}{(x+2)(x+3)} = \frac{A}{(x+2)} + \frac{B}{(x+3)}$$

$$\frac{x}{(x+2)(x+3)} = \frac{(-2)}{(x+2)} + \frac{3}{(x+3)}$$

$$I = \int \frac{x}{(x+2)(x+3)} dx = \int \left[\frac{(-2)}{(x+2)} + \frac{3}{(x+3)}\right] dx$$

$$= \int \frac{(-2)}{(x+2)} dx + \int \frac{3}{(x+3)} dx$$

$$= (-2) \int \frac{1}{(1x+2)} dx + 3 \int \frac{1}{(1x+3)} dx$$

$$\int \frac{1}{ax+b} dx = \log|ax+b| \cdot \frac{1}{a} + C$$

$$I = (-2) \cdot \log|(x+2)| \cdot \frac{1}{1} + 3 \cdot \log|x+3| \cdot \frac{1}{1} + C$$

$$I = (-2) \cdot \log|x+2| + 3 \cdot \log|x+3| + C$$

Q. $I = \int \frac{1}{x+5} dx$

$\int \frac{1}{ax+b} dx = \log|ax+b| \cdot \frac{1}{a} + c$

$I = \log|x+5| \cdot \frac{1}{1} + c$

$I = \log|x+5| + c$

Q. $I = \int \frac{x}{x+5} \cdot dx$

$I = \int \frac{(x+5)-5}{(x+5)} dx$

$= \int \left[\cancel{\frac{(x+5)}{(x+5)}} - \frac{5}{(x+5)} \right] dx$

$= \int \left[1 - \frac{5}{(x+5)} \right] dx$

$= \int 1 \cdot dx - \int \frac{5}{(x+5)} dx$

$= x - 5 \int \frac{1}{x+5} \cdot dx + c$

$= x - 5 \cdot \log|x+5| \cdot \frac{1}{1} + c$

$I = x - 5 \cdot \log|x+5| + c$ //

$I = \int \frac{x}{x^2+5} dx$

$f(x) = x^2+5$

diff. w.r.t x

$f'(x) = 2x + 0$

$= 2x$

$I = \frac{1}{2} \int \frac{2x}{x^2+5} dx$

$\int \frac{f'(x)}{f(x)} dx = \log|f(x)| + c$

$\therefore I = \frac{1}{2} \cdot \log|x^2+5| + c$ //

Third term sum

Q. $I = \int \frac{dx}{1x^2+4x+25}$

First term = FT = 1

Middle term = MT = 4

Third Term $= \frac{(MT)^2}{4 \times FT}$

$= \frac{4^{\cancel{2}}}{\cancel{4} \times 1}$

Third term $= 4$

$\therefore I = \int \frac{dx}{x^2 + 4x + 4 - 4 + 25}$

$a^2 = x^2$ $\quad$ $b^2 = 4$

$a = x$ $\quad$ $b = 2$

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$\therefore I = \int \frac{dx}{(x+2)^2 + 21}$

$\int \frac{1}{x^2+a^2} dx = \frac{1}{a} \cdot \tan^{-1}\left(\frac{x}{a}\right) + c$

$x = (x+2)$ $\quad$ $a^2 = 21$

$a = \sqrt{21}$

$\therefore I = \frac{1}{\sqrt{21}} \cdot \tan^{-1}\left(\frac{x+2}{\sqrt{21}}\right) + C$ //

U*V Sum

$$I = \int x \cdot \tan^{-1} x \, dx$$

$$\int u \cdot v \, dx = u \int v \, dx - \int \left[\frac{d}{dx} u \cdot \int v \, dx \right] dx$$

L =
I = $\tan^{-1} x \Rightarrow u$
A = $x \Longrightarrow v$
T =
E =

$$\therefore I = \int \tan^{-1} x \cdot x \cdot dx = \tan^{-1} x \int x \cdot dx - \int \left[\frac{d}{dx} \tan^{-1} x \cdot \int x \cdot dx \right] dx$$

$$= \tan^{-1} x \cdot \left[\frac{x^2}{2} \right] - \int \left[\frac{1}{1+x^2} \cdot \frac{x^2}{2} \right] \cdot dx + C$$

$$= \tan^{-1} x \cdot \left(\frac{x^2}{2} \right) - \frac{1}{2} \int \left[\frac{x^2}{1+x^2} \right] \cdot dx + C$$

$$= \frac{x^2}{2} \cdot \tan^{-1} x - \frac{1}{2} \int \left[\frac{(1+x^2) - 1}{(1+x^2)} \right] dx + C$$

$$= \frac{x^2}{2} \cdot \tan^{-1} x - \frac{1}{2} \int \left[\frac{\cancel{(1+x^2)}}{\cancel{(1+x^2)}} - \frac{1}{(1+x^2)} \right] dx + C$$

$$= \frac{x^2}{2} \cdot \tan^{-1} x - \frac{1}{2} \int \left[1 - \frac{1}{(1+x^2)} \right] dx + C$$

$$= \frac{x^2}{2} \cdot \tan^{-1} x - \frac{1}{2} \left[\int 1 \cdot dx - \int \frac{1}{1+x^2} \cdot dx \right] + C$$

$$I = \frac{x^2}{2} \cdot \tan^{-1} x - \frac{1}{2} \left[x - \tan^{-1} x \right] + C \quad //$$

$$I = \int \log x \, dx$$

$$\Rightarrow \quad I = \int 1 \cdot \log x \, dx$$

$$\int u \cdot v \, dx = u \int v \, dx - \int \left[\frac{d}{dx} u \cdot \int v \, dx \right] dx$$

L $= \log x = u$

I

A $= 1 \Rightarrow v$

T

E

$$\int \log x \cdot 1 \, dx = \log x \int 1 \cdot dx - \int \left[\frac{d}{dx} \log x \cdot \int 1 \, dx \right] dx$$

$$= \log x \, [x] - \int \left[\frac{1}{\cancel{x}} \cdot \cancel{x} \right] dx + C$$

$$= x \cdot \log x - \int 1 \cdot dx + C$$

$$I = x \cdot \log x - x + C$$

$$I = \int x \cdot e^x \, dx$$

JM definite Integration

$$I = \int_{a=2}^{b=5} \frac{\sqrt{x}}{\sqrt{7-x}+\sqrt{x}}\,dx \quad \text{——— (1)}$$

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$(x) \Rightarrow (a+b-x)$$

$$(x) \Rightarrow (2+5-x) = (7-x)$$

$\therefore$ eqn (1) $\Rightarrow$

$$I = \int_2^5 \frac{\sqrt{7-x}}{\sqrt{7-(7-x)}+\sqrt{7-x}}\,dx$$

$$= \int_2^5 \frac{\sqrt{7-x}}{\sqrt{7-7+x}+\sqrt{7-x}}\,dx$$

$$I = \int_2^5 \frac{\sqrt{7-x}}{\sqrt{x}+\sqrt{7-x}}\,dx \quad \text{——— (2)}$$

$$I = \int_{a=2}^{b=5} \frac{\sqrt{x}}{\sqrt{7-x}+\sqrt{x}}\,dx \quad \text{——— (1)}$$

eqn (2) + (1)

$$I + I = \int_2^5 \frac{\sqrt{7-x}}{\sqrt{x}+\sqrt{7-x}}\,dx + \int_2^5 \frac{\sqrt{x}}{\sqrt{7-x}+\sqrt{x}}\,dx$$

$$2I = \int_2^5 \frac{\cancel{(\sqrt{7-x}+\sqrt{x})}^{1}}{\cancel{(\sqrt{x}+\sqrt{7-x})}_{1}}\,dx$$

$$2I = \int_2^5 1\,dx$$

$$2I = [x]_2^5$$

$$2I = [5-2]$$

$$2I = 3$$

$$I = \frac{3}{2}$$

Substitution Method

$$I = \int \frac{1}{2+3\cos 2x}\cdot dx$$

$\Rightarrow$

$$\boxed{\begin{aligned} &\tan x = t \\ &\cos 2x = \frac{1-t^2}{1+t^2} \\ &dx = \frac{1}{1+t^2}\,dt \\ &\sin 2x = \frac{2t}{1+t^2} \end{aligned}}$$

$$I = \int \frac{1}{2+3\left(\frac{1-t^2}{1+t^2}\right)}\cdot\frac{1}{(1+t^2)}\cdot dt$$

$$\therefore I = \int \frac{1}{\frac{2(1+t^2)+3(1-t^2)}{1\cancel{(1+t^2)}}}\cdot\frac{dt}{\cancel{(1+t^2)}}$$

$$\therefore I = \int \frac{1}{2(1+t^2)+3(1-t^2)}\cdot dt$$

$$= \int \frac{1}{2+2t^2+3-3t^2}\cdot dt$$

$$I = \int \frac{1}{5-t^2}\cdot dt$$

$$\int \frac{1}{a^2-x^2} = \frac{1}{2a}\cdot\log\left|\frac{a+x}{a-x}\right|+C$$

$a^2 = 5 \quad | \quad x^2 = t^2$

$a = \sqrt{5} \quad | \quad x = t$

$$\therefore I = \frac{1}{2\sqrt{5}}\cdot\log\left|\frac{\sqrt{5}+t}{\sqrt{5}-t}\right|+C$$

$$= \frac{1}{2\sqrt{5}}\cdot\log\left|\frac{\sqrt{5}+\tan x}{\sqrt{5}-\tan x}\right|+C$$

$$I = \int \frac{1}{2+3\cos x}\,dx$$

$\Rightarrow$

$$\boxed{\begin{aligned} &\tan(x/2) = t \\ &\cos x = \frac{1-t^2}{1+t^2} \\ &dx = \frac{2dt}{1+t^2} \\ &\sin x = \frac{2t}{1+t^2} \end{aligned}}$$

$$I = \int \frac{1}{2+3\cos x}\cdot dx = \int \frac{1}{2+3\left(\frac{1-t^2}{1+t^2}\right)}\cdot\frac{2dt}{(1+t^2)}$$

$$\therefore I = \int \frac{1}{\frac{2(1+t^2)+3(1-t^2)}{\cancel{(1+t^2)}}}\cdot\frac{2dt}{\cancel{(1+t^2)}}$$

$$= 2\int \frac{1}{2(1+t^2)+3(1-t^2)}\cdot dt$$

$$= 2\int \frac{1}{2+2t^2+3-3t^2}\,dt$$

$$= 2\int \frac{1}{5-t^2}\,dt$$

$$\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\cdot\log\left|\frac{a+x}{a-x}\right|+C$$

$a^2 = 5 \quad | \quad x^2 = t^2$

$\therefore a = \sqrt{5} \quad | \quad x = t$

$$\therefore I = \cancel{2}\left[\frac{1}{\cancel{2}\sqrt{5}}\cdot\log\left|\frac{\sqrt{5}+t}{\sqrt{5}-t}\right|\right]+C$$

$$I = \frac{1}{\sqrt{5}}\cdot\log\left|\frac{\sqrt{5}+\tan(x/2)}{\sqrt{5}-\tan(x/2)}\right|+C \;//$$

Substitution Method

$$I=\int \frac{1}{x(2-\log x)(2\log x-1)}\,dx$$

$\Rightarrow$

$\log x = t$

diff. w.r.t. 'x'

$\frac{d}{dx}\log x = \frac{d}{dx}t$

$\frac{1}{x} = \frac{dt}{dx}$

$\frac{1}{x}\cdot dx = dt$

$$I=\int \frac{1}{(2-t)(2t-1)}\cdot dt$$

$$\frac{1}{(2-t)(2t-1)} = \frac{A}{(2-t)} + \frac{B}{(2t-1)}$$

$$1 = A(2t-1) + B(2-t) \text{ ——— } (1)$$

$2t-1=0$, $2t=1$, $t=1/2$ $\}$ eqⁿ (1) $\Rightarrow$ $1 = A[\cancel{2}\times\frac{1}{\cancel{2}}-1] + B(2-\frac{1}{2})$

$1 = A(0)+B\left(\frac{3}{2}\right)$

$1 = B\left(\frac{3}{2}\right) \Rightarrow \boxed{B=\frac{2}{3}}$

$2-t=0$, $2=t$ $\}$ eqⁿ (1) $\Rightarrow$ $1 = A(2\times2-1)+B(2-2)$

$1 = A(3)+B(0)$

$1 = A(3)$

$\therefore \boxed{A=\frac{1}{3}}$

$$\frac{1}{(2-t)(2t-1)} = \frac{1/3}{(2-t)} + \frac{2/3}{(2t-1)}$$

$$\therefore I=\int \frac{1}{(2-t)(2t-1)}dt$$

$$=\int\left[\frac{1/3}{(2-t)}+\frac{2/3}{(2t-1)}\right]dt$$

$$=\int\frac{1/3}{(2-t)}dt+\int\frac{2/3}{(2t-1)}dt$$

$$=\frac{1}{3}\int\frac{1}{2-t}dt+\frac{2}{3}\int\frac{1}{2t-1}dt$$

$$\int\frac{1}{ax+b}\cdot dx = \log|ax+b|\cdot\frac{1}{a}+C$$

$$\therefore I = \frac{1}{3}\cdot\log|2-t|\cdot\frac{1}{(-1)}+\frac{\cancel{2}}{3}\cdot\log|2t-1|\cdot\frac{1}{\cancel{2}}+C$$

$$I = -\frac{1}{3}\cdot\log|2-t|+\frac{1}{3}\log|2t-1|+C \;//$$

$$\therefore I = -\frac{1}{3}\cdot\log|2-\log x|+\frac{1}{3}\log|2\log x-1|+C \;//$$

Q. $$I=\int\frac{e^x}{(e^x-1)(e^x+1)}\,dx$$

$e^x = t$

differentiating w.r.t 'x'

$\frac{d}{dx}e^x = \frac{d}{dx}t$

$e^x = \frac{dt}{dx}$

$e^x\cdot dx = dt$

$$\therefore I = \int\frac{1}{(t-1)(t+1)}\cdot dt$$

$$\frac{1}{(t-1)(t+1)} = \frac{A}{(t-1)}+\frac{B}{(t+1)}$$

$$1 = A(t+1)+B(t-1) \text{ ——— } (1)$$

$t+1=0$, $t=-1$ $\}$ eqⁿ (1) $\Rightarrow$ $1 = A[(-1)+1]+B[(-1)-1]$

$1 = A(0)+B(-2)$

$1 = B(-2)$

$\frac{1}{(-2)} = B \Rightarrow \boxed{B=-1/2}$

$t-1=0$, $t=1$ $\}$ eqⁿ (1) $\Rightarrow$ $1 = A(1+1)+B(1-1)$

$1 = A(2)+B(0)$

$1 = A(2)$

$\boxed{\frac{1}{2}=A}$

$$= \frac{A}{(t-1)}+\frac{B}{(t+1)}$$

$$\frac{1}{(t-1)(t+1)} = \frac{1/2}{(t-1)}+\frac{(-1/2)}{(t+1)}$$

$$\therefore I = \int\frac{1}{(t-1)(t+1)}\cdot dt$$

$$=\int\left[\frac{1/2}{(t-1)}+\frac{(-1/2)}{(t+1)}\right]dt$$

$$=\int\frac{1/2}{(t-1)}dt+\int\frac{-1/2}{(t+1)}dt$$

$$=\frac{1}{2}\int\frac{1}{(t-1)}dt+\left(\frac{-1}{2}\right)\int\frac{1}{(t+1)}dt$$

$$\int\frac{1}{ax+b}dx = \log|ax+b|\cdot\frac{1}{a}+C$$

$$\therefore I = \frac{1}{2}\log|t-1|\cdot\frac{1}{1}+\left(\frac{-1}{2}\right)\cdot\log|t+1|\cdot\frac{1}{1}+C \;//$$

$$=\frac{1}{2}\log|e^x-1|-\frac{1}{2}\log|e^x+1|+C$$

$$=\frac{1}{2}\left[\log|e^x-1|-\log|e^x+1|\right]+C$$

$$I = \frac{1}{2}\log\frac{|e^x-1|}{|e^x+1|}+C \;//$$

Substitution Method

$$I = \int \frac{e^{x}(x+1)}{\cos^2(x\cdot e^{x})}\,dx$$

$x\cdot e^{x} = t$

differentiating w.r.t 'x'

$$\frac{d}{dx}(\underset{u}{x}\cdot \underset{v}{e^{x}}) = \frac{d}{dx}t$$

$$u\cdot\frac{d}{dx}v + v\cdot\frac{d}{dx}u = \frac{dt}{dx}$$

$$x\cdot\frac{d}{dx}e^{x} + e^{x}\cdot\frac{d}{dx}x = \frac{dt}{dx}$$

$$x\cdot e^{x} + e^{x}\cdot 1 = \frac{dt}{dx}$$

$$e^{x}(x+1) = \frac{dt}{dx}$$

$$e^{x}(x+1)\cdot dx = dt$$

$$I = \int \frac{1}{\cos^2 t}\,dt$$

$$I = \int \sec^2 t\cdot dt$$

$$= \tan t + C$$

$$I = \tan(x\cdot e^{x}) + c$$

2 Marks

Q. find degree & order

$$\sqrt{\frac{d^2y}{dx^2}} - \frac{dy}{dx} - xy^2 = 0$$

$\Rightarrow$ Order = 2

$$\sqrt{\frac{d^2y}{dx^2}} - \frac{dy}{dx} - xy^2 = 0$$

$$\sqrt{\frac{d^2y}{dx^2}} = xy^2 + \frac{dy}{dx}$$

$$\left(\frac{d^2y}{dx^2}\right)^{1/2} = \left(xy^2 + \frac{dy}{dx}\right)$$

squaring both sides

$$\left[\left(\frac{d^2y}{dx^2}\right)^{\cancel{1/2}}\right]^{\cancel{2}} = \left(xy^2 + \frac{dy}{dx}\right)^2$$

$$\left(\frac{d^2y}{dx^2}\right)^1 = \left(xy^2 + \frac{dy}{dx}\right)^2$$

degree = 1

order = 2

Q. find Order & degree

$$\sqrt[3]{\frac{dy}{dx} + y} = \sqrt[4]{\frac{d^2y}{dx^2}}$$

$$\Rightarrow \left(\frac{dy}{dx} + y\right)^{1/3} = \left(\frac{d^2y}{dx^2}\right)^{1/4}$$

taking cube on both sides

$$\left[\left(\frac{dy}{dx} + y\right)^{1/\cancel{3}}\right]^{\cancel{3}} = \left[\left(\frac{d^2y}{dx^2}\right)^{1/4}\right]^3$$

$$\left(\frac{dy}{dx} + y\right)^1 = \left(\frac{d^2y}{dx^2}\right)^{3/4}$$

taking raise 4 both side

$$\left[\left(\frac{dy}{dx} + y\right)^1\right]^4 = \left[\left(\frac{d^2y}{dx^2}\right)^{3/\cancel{4}}\right]^{\cancel{4}}$$

$$\left(\frac{dy}{dx} + y\right)^1 = \left(\frac{d^2y}{dx^2}\right)^3$$

Order = 2

degree = 3

4 Marks

Q. If $y = A\sin x + B\cos x$ show that $\frac{d^2y}{dx^2} + y = 0$

$\Rightarrow$

$$y = A\sin x + B\cos x \quad \text{———} \ (1)$$

differentiating w.r.t x

$$\frac{d}{dx}y = \frac{d}{dx}(A\sin x + B\cos x)$$

$$= \frac{d}{dx}A\sin x + \frac{d}{dx}B\cos x$$

$$= A\cdot\frac{d}{dx}\sin x + B\cdot\frac{d}{dx}\cos x$$

$$= A(\cos x) + B\cdot(-\sin x)$$

$$\frac{dy}{dx} = A\cos x - B\sin x$$

differentiating w.r.t 'x'

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(A\cos x - B\sin x)$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}A\cos x - \frac{d}{dx}B\sin x$$

$$= A\frac{d}{dx}\cos x - B\cdot\frac{d}{dx}\sin x$$

$$= A(-\sin x) - B(\cos x)$$

$$\frac{d^2y}{dx^2} = -A\sin x - B\cos x$$

$$\frac{d^2y}{dx^2} = -(A\sin x + B\cos x)$$

$$\frac{d^2y}{dx^2} = -y$$

$$\frac{d^2y}{dx^2} + y = 0$$

// Hence proved//

4 Marks

Q. Remove Arbitrary Constant

$y^2 = 4ax$

$\Rightarrow$

$y^2 = 4ax$

differentiating w.r.t 'x'

$\frac{d}{dx} y^2 = \frac{d}{dx} 4ax$

$\frac{d}{dx} x^2 = 4a \cdot \frac{d}{dx} x$

$2x \cdot \frac{d}{dx} x = 4a(1)$

$2y \cdot \frac{dy}{dx} = 4a$ —— ①

$y^2 = 4ax$

$\frac{y^2}{x} = 4a$

$\therefore eq^n$ ① $\Rightarrow 2y \cdot \frac{dy}{dx} = \frac{y^2}{x}$

$2y \cdot \frac{dy}{dx} - \frac{y^2}{x} = 0$ //

$\therefore$ Required D.E

Q. Remove Arbitrary Constant

if $y = A \cdot \cos 3x + B \sin 3x$

$\Rightarrow$

$y = A \cdot \cos 3x + B \sin 3x$ —— ①

differentiating w.r.t x

$\frac{d}{dx} y = \frac{d}{dx}(A \cdot \cos 3x + B \sin 3x)$

$= \frac{d}{dx} A \cos 3x + \frac{d}{dx} B \sin 3x$

$= A \cdot \frac{d}{dx} \cos 3x + B \frac{d}{dx} \sin 3x$

$= A(-\sin x) \frac{d}{dx} x + B(\cos x) \cdot \frac{d}{dx} x$

$= A(-\sin 3x) \frac{d}{dx} 3x + B \cos 3x \cdot \frac{d}{dx} 3x$

$\frac{dy}{dx} = -A \sin 3x \cdot 3 \frac{d}{dx} x + B \cdot \cos 3x \cdot 3 \frac{d}{dx} x$

$\frac{dy}{dx} = -A \sin 3x \cdot 3(1) + B \cdot \cos 3x \cdot 3(1)$

$\frac{dy}{dx} = -3A \sin 3x + 3B \cdot \cos 3x$

differentiating w.r.t 'x'

$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(-3A \sin 3x + 3B \cdot \cos 3x)$

$\frac{d^2y}{dx^2} = \frac{d}{dx} -3A \cdot \sin 3x + \frac{d}{dx} 3B \cos 3x$

$\frac{d^2y}{dx^2} = -3A \cdot \frac{d}{dx} \sin 3x + 3B \frac{d}{dx} \cos 3x$

$= -3A \cdot \cos x \cdot \frac{d}{dx} x + 3B \cdot (-\sin x) \cdot \frac{d}{dx} x$

$= -3A \cdot \cos 3x \cdot \frac{d}{dx} 3x + 3B(-\sin 3x) \cdot \frac{d}{dx} 3x$

$\frac{d^2y}{dx^2} = -3A \cdot \cos 3x \cdot 3(1) - 3B \sin 3x \cdot 3(1)$

$\frac{d^2y}{dx^2} = -9A \cdot \cos 3x - 9B \sin 3x$

$\frac{d^2y}{dx^2} = -9(A \cos 3x + B \sin 3x)$ y — eq^n ①

$\frac{d^2y}{dx^2} = -9y$

$\frac{d^2y}{dx^2} + 9y = 0$ // Required DE //

4Marks

Q. Solve D.E

1) $\frac{dy}{dx} + y \cdot \cot x = \operatorname{cosec} x$

$\Rightarrow$

step ① $\frac{dy}{dx} + Py = Q$

$P = \cot x$

$Q = \operatorname{cosec} x$

step ② IF = Integrated factor

$= e^{\int P \cdot dx}$

$\int P \cdot dx = \int \cot x \cdot dx$

$\int P dx = \log|\sin x|$

$IF = e^{\int P \cdot dx}$

$IF = e^{\log_e \boxed{|\sin x|}}$

$e^{\log_e x} = x$

$\therefore IF = \sin x$

step ③. $y \cdot IF = \int Q \cdot IF \, dx + C$

$y \cdot \sin x = \int \operatorname{cosec} x \cdot \sin x \cdot dx + C$

$= \int \frac{1}{\cancel{\sin x}} \cdot \cancel{\sin x} \, dx + c$

$= \int 1 \cdot dx + c$

$y \cdot \sin x = x + c$

$y \cdot \sin x - x - c = 0$ //

Q. Solve D.E

2) $x \cdot \frac{dy}{dx} - y = x^2$ —— ①

$\Rightarrow$

Step ① $\frac{dy}{dx} + Py = Q$

dividing eq^n ① by 'x'

$\frac{\cancel{x} \cdot \frac{dy}{dx}}{\cancel{x}} - \frac{y}{x} = \frac{x^{\cancel{2}}}{\cancel{x}}$

$\frac{dy}{dx} - y \underbrace{x^{-1}}_{P} = \underbrace{x}_{Q}$

$P = x^{-1}$

$Q = x$

Step ② Find Integrated factor

$IF = e^{\int P \cdot dx}$

$\int P \cdot dx = \int x^{-1} \cdot dx$

$= \int \frac{1}{x} \cdot dx$

$\int P \, dx = \log x$

$\therefore IF = e^{\log x}$

$e^{\log X} = X$

$\therefore IF = x$

Step ③. $y \cdot IF = \int Q \cdot IF \, dx + C$

$y \cdot x = \int x \cdot x \cdot dx + c$

$xy = \int x^2 dx + c$

$xy = \frac{x^3}{3} + c$

$xy - \frac{x^3}{3} - c = 0$ //

Q. Solve $(1+x^2)\,dy - x^2y.dx = 0$

$\Rightarrow$

$$(1+x^2)\,dy - x^2y.dx = 0$$

$$\boxed{(1+x^2)}\,dy = x^2 y\,dx$$

$$\frac{dy}{y} = \frac{x^2}{1+x^2}\,dx$$

$$\frac{1}{y}.dy = \frac{x^2}{1+x^2}\,dx$$

Integrating..

$$\int \frac{1}{y}.dy = \int \frac{x^2}{1+x^2}\,dx$$

$$\log y + c = \int \frac{(1+x^2)-1}{(1+x^2)}\,dx$$

$$\log y + c = \int \left[\cancel{\frac{(1+x^2)}{(1+x^2)}} - \frac{1}{(1+x^2)}\right] dx$$

$$= \int \left[1 - \frac{1}{1+x^2}\right] dx$$

$$= \int 1.dx - \int \frac{1}{1+x^2}.dx$$

$$\log y + c = x - \tan^{-1}(x)$$

$$\log y + c - x + \tan^{-1}(x) = 0 \;//$$

2 Marks

Q. find area bounded by $y = x^2$ & X-axis from $x=0$ to $x=3$

$\Rightarrow$

$$\text{Area} = \int_a^b y \cdot dx$$

$$= \int_{a=0}^{b=3} x^2 \, dx$$

$$= \left[\frac{x^3}{3}\right]_0^3$$

$$= \left[\frac{3^3}{3} - \frac{0^3}{3}\right]$$

$$= \left[\frac{27}{3} - 0\right]$$

Area $= 9$ sq. units

Q. find area bounded by $y = \sin x$ & X-axis from $x=0$ to $x = \pi/2$

$\Rightarrow$

$$\text{Area} = \int_a^b y \cdot dx$$

$$= \int_{a=0}^{b=\pi/2} \sin x \cdot dx$$

$$= \left[-\cos x\right]_0^{\pi/2}$$

$$= -\cos\left(\frac{\pi}{2}\right) + \cos(0)$$

$$= -\cos(90^\circ) + \cos(0^\circ)$$

$$= 0 + 1$$

Area $= 1$

$= 1$ sq. units

$\pi = 180^\circ$

$\frac{\pi}{2} = \frac{\cancel{180}^{90}}{\cancel{2}_1} = 90^\circ$

Numerical Method

2M — 1) Bakhshali Method (k-scheme)

4M — 2) Bisection Method
3) Regula-falsi Method (false position Method)

4M / 6M — 4) Newton Raphson Method

4M / 6M — 5) Jacobi's Method

4M / 6M — 6) Gauss Siddel Method

Bakhshali Method

$\sqrt{5}$

step ① Consider N^2 whose Nearest exact squre root is possible with respect to 'S'

step ② $d = S - N^2$

step ③ $P = \frac{d}{2N}$

step ④ $A = N + P$

step ⑤ $\sqrt{S} = A - \frac{P^2}{2A}$

Q. find $\sqrt{9.2345}$

$\Rightarrow$ $S = 9.2345$

step ① $\Rightarrow$ $9 = 3^2 = N^2$

$\therefore N = 3$

step ② $d = S - N^2$

$= 9.2345 - 9$

$d = 0.2345$

step ③ $P = \frac{d}{2N} = \frac{0.2345}{(2 \times 3)}$

$= 0.03908$

$P \simeq 0.0391$

step ④

$A = N + P$

$= 3 + 0.0391$

$= 3.0391$

step ⑤ $\sqrt{S} = A - \frac{P^2}{2A}$

$= 3.0391 - \frac{(0.0391)^2}{(2 \times 3.0391)}$

$\sqrt{S} = 3.0388$

$\sqrt{9.2345} = 3.0388$

Bisection Method

Q. find real root of equation

$x^3 - x - 4 = \boxed{0}$ using Bisection Method

$\Rightarrow$

$f(x) = x^3 - x - 4$

if $x = 0$ $f(0) = 0^3 - 0 - 4$ $= -4 < 0$	if $x = 1$ $f(1) = 1^3 - 1 - 4$ $= -4 < 0$	if $x = 2$ $f(2) = 2^3 - 2 - 4$ $= 2 > 0$

variable	$a = 1$	$c = 1.5$	$d = 1.75$	$b = 2$
function	-4	-2.125	-0.3906 $\boxed{0}$	2

1st Iteration	**2nd Iteration**	**3rd Iteration**
$c = \frac{a+b}{2} = \frac{(1+2)}{2}$ $c = 1.5$ $f(c) = x^3 - x - 4$ $= 1.5^3 - 1.5 - 4$ $= -2.125 < 0$	$d = \frac{(c+b)}{2} = \frac{(1.5+2)}{2}$ $d = 1.75$ $f(d) = x^3 - x - 4$ $= 1.75^3 - 1.75 - 4$ $= -0.3906$	$e = \frac{(d+b)}{2}$ $= \frac{(1.75+2)}{2} = 1.875$ $f(e) = x^3 - x - 4$ $= 1.875^3 - 1.875 - 4$ $= 0.7168$

4M

$\therefore$ Root of function $x^3 - x - 4 = 0$ is $e = 1.875$ //

Regula-falsi Method

Q. find root of eq^n $x^3-4x-9=0$ in interval $(2,3)$ by using false position Method

$\Rightarrow$

$a=2$	$b=3$
$f(a)=x^3-4x-9$	$f(b)=x^3-4x-9$
$=2^3-4\times2-9$	$=3^3-4\times3-9$
$=-9<0$	$=6>0$

Variable	$a=2$	$c=2.6$	$d=2.693$		$b=3$
function	$f(a)=-9$	$f(c)=-1.824$	$f(d)=-0.242$	$\boxed{0}$	$f(b)=6$

1st Iteration

$$c=\frac{a.f(b)-b.f(a)}{f(b)-f(a)}$$

$$=\frac{2\times6-3\times(-9)}{6-(-9)}$$

$c=2.6$

$f(c)=x^3-4x-9$

$=2.6^3-4\times2.6-9$

$f(c)=-1.824$

2nd Iteration

$$d=\frac{c\,f(b)-b\,f(c)}{f(b)-f(c)}$$

$$=\frac{2.6\times6-3\times(-1.824)}{6-(-1.824)}$$

$d=2.693$

$f(d)=x^3-4x-9$

$=2.693^3-4\times2.693-9$

$f(d)=-0.242$ //

3rd Iteration

$$e=\frac{d\,f(b)-b.f(d)}{f(b)-f(d)}$$

$$e=\frac{2.693\times6-3\times(-0.242)}{6-(-0.242)}$$

$e=2.705$

$f(e)=x^3-4x-9$

$=2.705^3-4\times2.705-9$

$=-0.027$

$\therefore$ Root of eq^n $x^3-4x-9=0$ is $e=2.705$ //

Newton Raphson Method

Q. find $\sqrt[3]{100} = ?$ using Newton-Raphson Method

$\Rightarrow$

$x = \sqrt[3]{100}$

$x = (100)^{1/3}$

taking cube on both sides

$x^3 = [(100)^{1/3}]^3$

$x^3 = 100$

$f(x) = x^3 - 100 = 0$ —— ①

differentiating w.r.t 'x'

$f'(x) = \frac{d}{dx}(x^3 - 100)$

$= \frac{d}{dx}x^3 - \frac{d}{dx}100$

$= 3x^2 - 0$

$f'(x) = 3x^2$ —— ②

$f(x) = x^3 - 100$ —— ①

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

* $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$\boxed{\sqrt[3]{100} = 4.6415 \approx 4.642 \quad x_0 = 4}$

$\sqrt[3]{100} = (100)^{1/3}$

$= (100) \wedge (1 \div 3)$

$\cong 4.642$ //

1st Iteration $n = 0$, $x_0 = 4$

$x_{0+1} = x_0 - \frac{f(x_0)}{f'(x_0)}$

$x_1 = 4 - \frac{(-36)}{48}$

$x_1 = 4.75$

$f(x) = x^3 - 100$

$f(x_0) = x_0^3 - 100$

$= 4^3 - 100$

$= -36$

$f'(x) = 3x^2$

$f'(x_0) = 3 \times 4^2$

$= 48$

2nd Iteration $n = 1$, $x_1 = 4.75$

$x_{1+1} = x_1 - \frac{f(x_1)}{f'(x_1)}$

$x_2 = 4.75 - \frac{7.172}{67.688}$

$x_2 = 4.644$ //

$f(x) = x^3 - 100$

$f(x_1) = x_1^3 - 100$

$= 4.75^3 - 100$

$= 7.172$

$f'(x) = 3x^2$

$f'(x_1) = 3 \times x_1^2$

$= 3 \times 4.75^2$

$= 67.688$

3rd Iteration $n = 2$, $x_2 = 4.644$

$x_{2+1} = x_2 - \frac{f(x_2)}{f'(x_2)}$

$x_3 = 4.644 - \frac{0.156}{64.7}$

$= 4.6416$

$x_3 \simeq 4.642$ /

$f(x) = x^3 - 100$

$f(x_2) = x_2^3 - 100$

$= 4.644^3 - 100$

$= 0.156$

$f'(x) = 3x^2$

$f'(x_2) = 3\, x_2^2$

$= 3 \times 4.644^2$

$= 64.7$

$\therefore \sqrt[3]{100} = 4.642$ //

Jacobi Method

Q. $2x + 10y + z = 13$ —— ①

$10x + y + 2z = 12$ —— ②

$2x + 2y + 10z = 14$ —— ③

⇒ Coeff. of 'x' is Max. in eqn ②

$10x + y + 2z = 12$

$10x = 12 - y - 2z$

$x = \frac{(12 - y - 2z)}{10}$

Coeff. of 'y' is Max. in eqn ①

$2x + 10y + z = 13$

$10y = 13 - 2x - z$

$y = \frac{(13 - 2x - z)}{10}$

eqn ③ 'z'

$2x + 2y + 10z = 14$

$10z = 14 - 2x - 2y$

$z = \frac{(14 - 2x - 2y)}{10}$

1st Iteration

$x_0 = 0,\ y_0 = 0,\ z_0 = 0$

$x_1 = \frac{(12 - y - 2z)}{10} = \frac{(12 - 0 - 2\times 0)}{10}$

$x_1 = 1.2$

$y_1 = \frac{(13 - 2x - z)}{10} = \frac{(13 - 2\times 0 - 0)}{10}$

$y_1 = 1.3$

$z_1 = \frac{(14 - 2x - 2y)}{10} = \frac{(14 - 2\times 0 - 2\times 0)}{10}$

$z_1 = 1.4$

$x_1 = 1.2,\ y_1 = 1.3,\ z_1 = 1.4$

2nd Iteration

$x_1 = 1.2,\ y_1 = 1.3,\ z_1 = 1.4$

$x_2 = \frac{(12 - y - 2z)}{10} = \frac{(12 - 1.3 - 2\times 1.4)}{10}$

$x_2 = \frac{7.9}{10} = 0.79$

$y_2 = \frac{(13 - 2x - z)}{10} = \frac{(13 - 2\times 1.2 - 1.4)}{10}$

$y_2 = 0.92$

$z_2 = \frac{(14 - 2x - 2y)}{10} = \frac{(14 - 2\times 1.2 - 2\times 1.3)}{10}$

$z_2 = 0.9$

$x_2 = 0.79,\ y_2 = 0.92,\ z_2 = 0.9$

3rd Iteration

$x_2 = 0.79,\ y_2 = 0.92,\ z_2 = 0.9$

$x_3 = \frac{(12 - y - 2z)}{10} = \frac{(12 - 0.92 - 2\times 0.9)}{10}$

$= \frac{9.28}{10} = 0.928$

$y_3 = \frac{(13 - 2x - z)}{10} = \frac{(13 - 2\times 0.79 - 0.9)}{10}$

$= \frac{10.52}{10} = 1.052$

$z_3 = \frac{(14 - 2x - 2y)}{10} = \frac{(14 - 2\times 0.79 - 2\times 0.92)}{10}$

$= \frac{10.58}{10} = 1.058$ //

$$\boxed{\begin{aligned} \text{Solution}\quad x_3 &= 0.928 \\ y_3 &= 1.052 \\ z_3 &= 1.058 \end{aligned}}$$

Gauss-Seidel Method

Q $x+4y+2z=15$ —— ①

$x+2y+5z=20$ —— ②

$5x+2y+z=12$ —— ③

⇒ Coeff. of 'x' is max. in eqⁿ ③

$5x+2y+z=12$

$5x=12-2y-z$

$x=\dfrac{(12-2y-z)}{5}$

Coeff. of y is Max. in eqⁿ ①

$x+4y+2z=15$

$4y=15-x-2z$

$y=\dfrac{(15-x-2z)}{4}$

eqⁿ ②

$x+2y+5z=20$

$5z=20-x-2y$

$z=\dfrac{(20-x-2y)}{5}$

1st Iteration

$x_0=0,\ y_0=0,\ z_0=0$

$x_1=\dfrac{(12-2y-z)}{5}=\dfrac{(12-2\times0-0)}{5}$

$x_1=2.4$

$x_1=2.4,\ y_0=0,\ z_0=0$

$y_1=\dfrac{(15-x-2z)}{4}=\dfrac{(15-2.4-2\times0)}{4}$

$y_1=3.15$

$x_1=2.4,\ y_1=3.15,\ z_0=0$

$z_1=\dfrac{(20-x-2y)}{5}=\dfrac{(20-2.4-2\times3.15)}{5}$

$z_1=2.26$

$x_1=2.4,\ y_1=3.15,\ z_1=2.26$

2nd Iteration

$x_1=2.4,\ y_1=3.15,\ z_1=2.26$

$x_2=\dfrac{(12-2y-z)}{5}=\dfrac{(12-2\times3.15-2.26)}{5}$

$x_2=0.688,\ y_1=3.15,\ z_1=2.26$

$y_2=\dfrac{(15-x-2z)}{4}=\dfrac{(15-0.688-2\times2.26)}{4}$

$y_2=2.448$

$x_2=0.688,\ y_2=2.448,\ z_1=2.26$

$z_2=\dfrac{(20-x-2y)}{5}=\dfrac{(20-0.688-2\times2.448)}{5}$

$z_2=2.8832$

$\therefore\ x_2=0.688,\ y_2=2.448,\ z_2=2.883$

PROBABLITY

14 Marks

4M 1) Binomial distribution

4M 2) Poisson's distribution

6M $\Leftarrow$ 3) Normal distribution

if $n < 10 \Rightarrow$ Binomial distribution

$n > 10 \Rightarrow$ Poisson's distribution

Binomial Distribution

$$P(X=r) = {}^nC_r \cdot p^r \cdot (1-p)^{n-r}$$

r = No. of successful events

n = Total No. of events

p = Probablity of one event success

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

$0! = 1$

$1! = 1$

$2! = 2 \times 1 = 2$

$3! = 3 \times 2 \times 1 = 6$

$4! = 4 \times 3 \times 2 \times 1 = 24$

$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

Mean = $n \cdot p$

$S.D = \sqrt{np(1-p)} = \sqrt{npq}$

Variance = $S.D^2 = npq$

Coefficient of Variation $= \sqrt{\dfrac{(1-p)}{np}}$ //

Q. In 200 sets of tosses 5 fair coins, in how many ways you can expect

a) Exact 2 heads
b) At least 2 heads
c) At Most 2 heads
d) Less than 2 heads
e) More than 2 heads.

⇒

$$P(X=r) = {}^nC_r \cdot p^r \cdot (1-p)^{n-r}$$

$n = 5$

$p = 0.5$

H — $1/2 = 0.5$

T — $1/2 = 0.5$

a) Exact 2 heads ⇒ $r=2$

$$P(X=2) = {}^5C_2 \cdot (0.5)^2 \cdot (1-0.5)^{5-2}$$

$= 10 \times 0.25 \times 0.125$

$= 0.3125$

∴ No. of ways $= 200 \times 0.3125$

$= 62.5$

$\simeq 63$ ways (say)

b) At least 2 heads ⇒ $r=2$

$$P(X \geq r) = P(X=2) + P(X=3) + P(X=4) + P(X=5)$$

$= {}^5C_2 \cdot (0.5)^2 (1-0.5)^{5-2} = 0.3125$

$+ {}^5C_3 \cdot (0.5)^3 (1-0.5)^{5-3}$ $+ 0.3125$

$+ {}^5C_4 (0.5)^4 \cdot (1-0.5)^{5-4}$ $+ 0.15625$

$+ {}^5C_5 (0.5)^5 (1-0.5)^{5-5}$ $+ 0.03125$

$= 0.8125$

∴ No. of ways $= 200 \times 0.8125$

$= 162.5$

$\simeq 163$ ways (say)

c) At Most 2 heads ($r=2$)

$$P(X \leq 2) = P(X=2) + P(X=1) + P(X=0)$$

$= {}^5C_2 \cdot (0.5)^2 \cdot (1-0.5)^{5-2} = 0.3125$

$+ {}^5C_1 (0.5)^1 \cdot (1-0.5)^{5-1}$ $+ 0.15625$

$+ {}^5C_0 (0.5)^0 \cdot (1-0.5)^{5-0}$ $+ 0.03125$

$= 0.5$

∴ No. of Ways $= 200 \times 0.5$

$= 100$ ways //

d) Less than 2 heads ⇒ $r=2$

$$P(X<2) = P(X=0) + P(X=1)$$

$= {}^5C_0 \cdot (0.5)^0 \cdot (1-0.5)^{5-0} = 0.03125$

$+ {}^5C_1 \cdot (0.5)^1 \cdot (1-0.5)^{5-1}$ $+ 0.15625$

$= 0.1875$

No. of ways $= 200 \times 0.1875$

$= 37.5$

$\simeq 38$ ways (say)

e) More than 2 heads

$$P(X>2) = P(X=3) + P(X=4) + P(X=5)$$

$= {}^5C_3 \cdot (0.5)^3 \cdot (1-0.5)^{5-3} = 0.3125$

$+ {}^5C_4 \cdot (0.5)^4 \cdot (1-0.5)^{5-4}$ $+ 0.15625$

$+ {}^5C_5 \cdot (0.5)^5 \cdot (1-0.5)^{5-5}$ $+ 0.03125$

$= 0.5$

∴ No. of ways $= 200 \times 0.5$

$= 100$ ways //

Poisson's distribution

if $n > 10 \Rightarrow$ Poisson's distribution
$n < 10 \Rightarrow$ Binomial distribution

$$P(X=r) = \frac{e^{-m} \cdot m^{r}}{r!}$$

r = No. of successful events
m = Mean = np

Variance = Mean = m

S.D = $\sqrt{m} = \sqrt{np}$

Q. If 2% of electric bulb manufactured are defective find probablity in sample of 100 bulbs

a) 3 bulbs defective

b) At most 3 bulbs defective

c) At least 3 bulbs defective

d) Less than 3 bulbs defective

e) More than 3 bulbs defective

⇒

$$P(X=r) = \frac{e^{-m} \cdot m^{r}}{r!}$$

$$2\% = p = \frac{2}{100} = 0.02$$

$$m = \text{mean} = np$$
$$= 100 \times 0.02$$
$$m = 2$$

a) 3 bulb defective $(r=3)$

$$P(X=3) = \frac{e^{-2} \cdot 2^{3}}{3!} = 0.1804$$

b) At Most 3 bulbs are defective

$$P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$$

$$= \frac{e^{-2} \cdot 2^{0}}{0!} + \frac{e^{-2} \cdot 2^{1}}{1!} + \frac{e^{-2} \cdot 2^{2}}{2!} + \frac{e^{-2} \cdot 2^{3}}{3!} = 0.1353 + 0.2707 + 0.2707 + 0.1804$$

$$= 0.8571$$

c) At least 3 bulbs defective

$$P(X \geq 3) = P(X=3) + P(X=4) + \cdots + P(X=100)$$

$$= 1 - \{P(X=2) + P(X=1) + P(X=0)\}$$

$$= 1 - \left\{ \frac{e^{-2} \cdot 2^{2}}{2!} + \frac{e^{-2} \cdot 2^{1}}{1!} + \frac{e^{-2} \cdot 2^{0}}{0!} \right\} = 1 - \{0.2707 + 0.2707 + 0.1353\}$$

$$= 0.3233$$

d) Less than 3 bulb defective

$$P(X<3) = P(X=0) + P(X=1) + P(X=2)$$

$$= \frac{e^{-2} \cdot 2^{0}}{0!} + \frac{e^{-2} \cdot 2^{1}}{1!} + \frac{e^{-2} \cdot 2^{2}}{2!} = 0.1353 + 0.2707 + 0.2707$$

$$= 0.6767$$

e) More than 3 bulbs defective

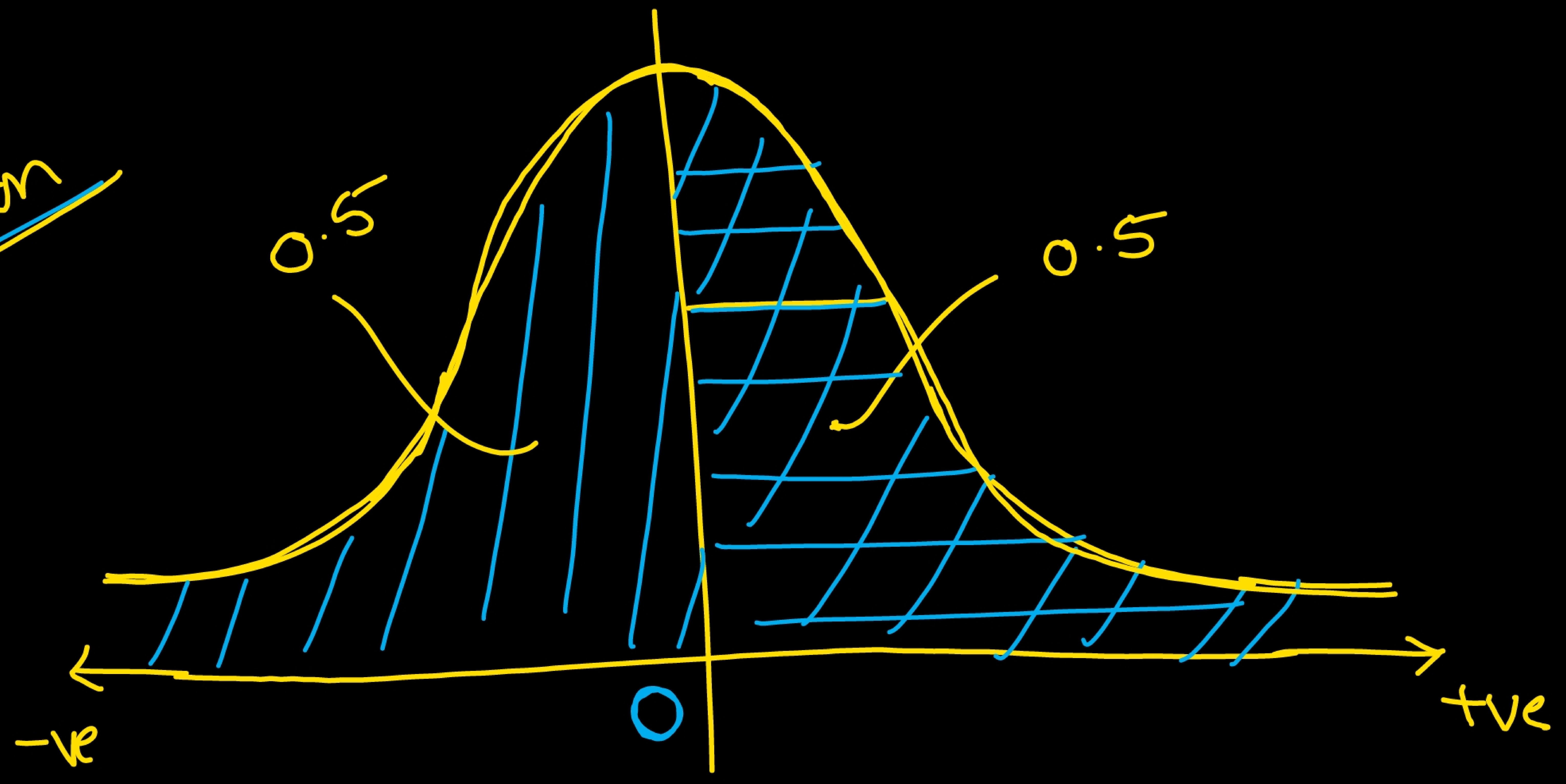
$$P(X>3) = P(X=4) + P(X=5) + \cdots + P(X=100)$$

$$= 1 - \{P(X=3) + P(X=2) + P(X=1) + P(X=0)\}$$

$$= 1 - \left\{ \frac{e^{-2} \cdot 2^{3}}{3!} + \frac{e^{-2} \cdot 2^{2}}{2!} + \frac{e^{-2} \cdot 2^{1}}{1!} + \frac{e^{-2} \cdot 2^{0}}{0!} \right\} = 1 - \{0.1804 + 0.2707 + 0.2707 + 0.1353\}$$

$$= 0.1429$$

Normal Distribution



$$Z = \frac{x - \mu}{\sigma}$$

μ = Mean

σ = S.D

1. Inverted bell shape curve
2. Area under the curve - Right side of origin = 0.5
3. — " ——— - Left side of origin = 0.5

Q. In test of 2000 electric bulb, its found that avg. life of bulb is 2040 hrs & std. deviation of 60 hrs. Estimate no. of bulbs likely to burn for - $A(2) = 0.4772$ & $A(1.833) = 0.4664$

a) More than 2150 hrs
b) Less than 2150 hrs
c) Between 1920 hrs & 2150 hrs
d) More than 1920 hrs
e) Less than 1920 hrs
f) Less than 1920 & More than 2150 hrs

⇒

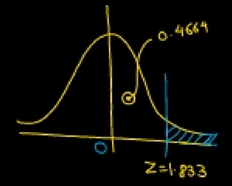
$\mu = \text{Mean} = 2040$ hrs

$\sigma = S.D = 60$ hrs

a) More than 2150 hrs ⇒ $x = 2150$

$$Z = \frac{x-\mu}{\sigma} = \frac{(2150-2040)}{60}$$

$$Z = \frac{110}{60} = 1.833$$



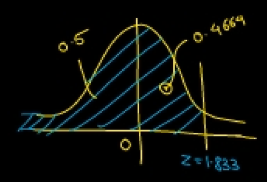
$\therefore P = 0.5 - 0.4664$
$= 0.0336$

$\therefore$ No. of bulbs having life more than 2150 hrs $= 0.0336 \times 2000$
$= 67.2$
$\simeq 67$ bulbs (say)

b) Less 50 hrs ($x = 2150$)

$$Z = \frac{\mu}{\sigma} = \frac{2150-2040}{60}$$

$$Z = \frac{110}{60} = 1.833$$



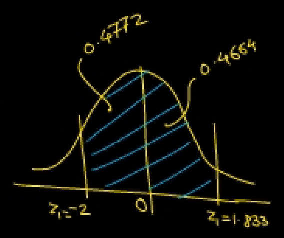
$\therefore P = 0.5 + 0.4664$
$= 0.9664$

$\therefore$ No. of bulbs having life less than 2150 hrs $= 0.9664 \times 2000$
$= 1932.8$
$\simeq 1934$ bulbs (say)

c) between 1920 hrs & 2150 hrs (x_1, x_2)

$Z_1 = \frac{x_1-\mu}{\sigma} = \frac{1920-2040}{60}$ | $Z_2 = \frac{x_2-\mu}{\sigma} = \frac{2150-2040}{60}$

$Z_1 = \frac{-120}{60} = -2$ | $= \frac{110}{60} = 1.833$



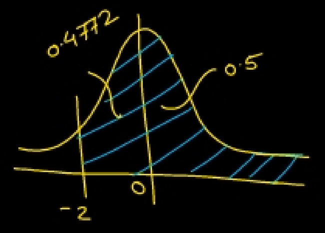
$\therefore P = 0.4772 + 0.4664$
$= 0.9436$

$\therefore$ No. of bulbs having life between 1920 & 2150 hrs $= 2000 \times 0.9436$
$= 1887.2$
$\simeq 1887$ bulbs (say)

d) More than 1920 hrs (x)

$$Z = \frac{x-\mu}{\sigma} = \frac{1920-2040}{60}$$

$$= \frac{-120}{60} = -2$$



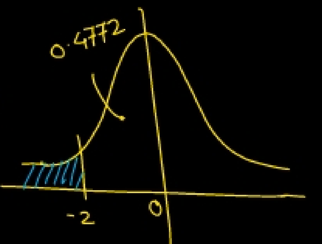
$P = 0.4772 + 0.5$
$= 0.9772$

$\therefore$ No. of bulbs having life More than 1920 hrs $= 2000 \times 0.9772$
$= 1954.4$
$\simeq 1954$ bulbs (say)

e) Less than 1920 hrs (x)

$$Z = \frac{x-\mu}{\sigma} = \frac{1920-2040}{60}$$

$$Z = \frac{-120}{60} = -2$$



$P = 0.5 - 0.4772$
$= 0.0228$

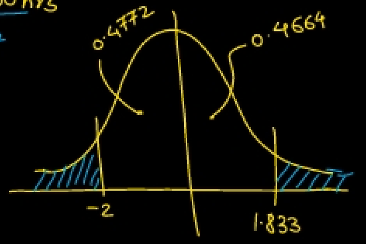
$\therefore$ No. of bulbs having life less than 1920 hrs $= 2000 \times 0.0228$
$= 45.6$
$\simeq 46$ bulbs (say)

f) Less than 1920 & More than 2150 hrs (x_1, x_2)

$Z_1 = \frac{x_1-\mu}{\sigma}$ | $Z_2 = \frac{x_2-\mu}{\sigma}$

$= \frac{1920-2040}{60}$ | $= \frac{2150-2040}{60}$

$Z_1 = \frac{-120}{60} = -2$ | $= \frac{110}{60} = 1.833$



$P = (0.5 - 0.4772)$
$+ (0.5 - 0.4664)$

$= 0.0564$

$\therefore$ No. of bulbs having life less than 1920 hrs & More than 2150 hrs $= 2000 \times 0.0564$
$= 112.8$
$\simeq 113$ bulbs (say)